MMM		HHH HHH HHH HHH HHH HHH HHH HHH HHH HH	RRRRRRRRRRRRRRRRRRRRRRRRRRRRRRRRRRRRRR		LLL LLL LLL LLL LLL LLL LLL LLL LLL LL
MMM MMM	††† †††	HHH HHH HHH HHH	RRR RRR	111 111 111	

000000 00 00 00 00		\$	PPPPPPPPPPPPPPPPPPPPPPPPPPPPPPPPPPPPPP	000000 00 00 00 00	HH HH HH HH HH HH HH HH HH HH HH HH HHHHHHHH
		\$			

FILEID**OTSPOWHH

- REAL*16 ** REAL*16 power routine 2 OTS\$POWHH Table of contents 16-SEP-1984 02:00:37 VAX/VMS Macro V04-00 Page 0 HISTORY ; Detailed current edit
DECLARATIONS
OTS\$POWHH_R3 - H_floating ** H_floating ; Detailed current edit history

2-0

16-SEP-1984 02:00:37 VAX/VMS Macro V04-00 6-SEP-1984 11:28:21 [MTHRTL.SRCJOTSPOWHH.MAR;1

Page (1)

2-0

.TITLE OTS\$POWHH - REAL*16 ** REAL*16 power routine .IDENT /2-006/ ; File: OTSPOWHH.MAR EDIT: JCW2006

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: FACILITY: Compiled code support library

: ABSTRACT:

0000

16

3333456789

40

H_floating base to H_floating power

VERSION: 2

AUTHOR:

Bob Hanek, 9-Mar-83; Version 2

Sym

OTS

AACAS 12FACCPPPPDGGHHHH NOTES 146 BODDT

PSE SAB

---Ini Com Pas Sym Pas Syn

0TS\$POWHH 2-006 OTS

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88

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MAC

```
G 2
0TS$POWHH
2-006
                                                                                   - REAL*16 ** REAL*16 power routine
                                                                                                                                                                                                                                                    VAX/VMS Macro V04-00
[MTHRTL.SRC]OTSPOWHH.MAR; 1
                                                                                  DECLARATIONS
                                                                                                0000
                                                                                                                   96
                                                                                                                                PSECT DECLARATIONS:
                                                                                                0000
                                                                                     00000000
                                                                                                                                                 .PSECT _OTS$CODE
                                                                                                                                                                                                              PIC, SHR, QUAD, EXE, NOWRT
                                                                                                0000
                                                                                                                                                                                                                                   ; program section for OTS$ code
                                                                                                0000
                                                                                                0000
                                                                                                                                CONSTANTS:
                                                                                                0000
                                                                                                0000
                                                                                                0000
                                                                                                                                The INDEX table gives the offset (in quadwords) to the appropriate entries in A1_TABLE and A2_TABLE. (NOTE: Entry 1 of the INDEX table is a special encoding that is intended to access the octawords immediately BEFORE the A1 and A2 tables.)
                                                                                                0000
                                                                                                0000
                                                                                                                 108
                                                                                                0000
                                                                                                0000
                                                                                                                 110
                                                                                               *XFF,
*X06,
*X0C,
*X10,
                               06 04
10 0E
14 14
18 18
12 26 28
20 34
36 36
36 36
36 36
36 36
36 36
36 36
36 36
                                              04AE48C048C026AC0
                                                                                                                            INDEX:
                                                                                                                                                BYTE.
                                                                                                                                                                                                   *X08.

*X16.

*X16.

*X126.

*X226.

*X338.

*X338.
                                                     048E28C048CE268CE
                                                              008E2668CE268CE
                                                                     080016AE26AE248AE
                                                                                    00001111226AE048AE
                                                                                                                                                                    *X06.

*X10.

*X14.

*X122.

*X226.

*X233.

*X338.

*X338.
                                                                                                                                                                                                                                                 *X0A.
*X0E.
*X1E.
*X1E.
*X2E.
*X2E.
*X2E.
*X3E.
*X3E.
                                                                             000106AE26AE048AE
                                                                                                                                                 .BYTE
                                                                                                                                                 .BYTE
                                                                                                                                                                                   *X16,
*X16,
*X1E,
*X226,
*X226,
*X334,
*X334,
                                                                                                                                                                                                                   *X16.

*X20.

*X224.

*X224.

*X224.

*X224.

*X236.

*X336.

*X336.

*X336.

*X336.
                                                                                                                                                 .BYTE
                                                                                                                                                 .BYTE
                                                                                                                 11122234567890123456789012345456789012
111122234567890123456789012345456789012
                                                                                                                                                 .BYTE
                                                                                                                                                 . ALIGN
                                                                                                                                                                    QUAD
                                                                                                                               for k=0,1,\ldots,32, the k-th entry of A1_TABLE is value of 2^{(k/32)} rounded to 113 fraction bits and the k-th entry of A2_TABLE is the value of 2(k/32) - A1_TABLE(k) rounded to 113 bits. For k=-1, A1_TABLE gives the value 2^{(1/64)} rounded to 113 bits and A2_TABLE give 2^{(1/64)} - A1_TABLE(-1) rounded to 113 bits
7A2ACA4F F7CA0EE6 7806A3E7 02C94001
                                                                                                                                                                    ^X7A2ACA4FF7CA0EE67806A3E702C94001
                                                                                                                                                 .OCTA
                                                                                                                           A1_TABLE
                                                                                                                                                                   00004001
05984001
08554001
11304001
17284001
10484001
23874001
29E94001
306F4001
371A4001
3DEA4001
                                              00000000
58570D31
989086CF
25851D01
D51783C7
889A7316
5623A6E7
00000000
8CA48EB6
42AAB718
                     00000000
7C5443AE
8B92F629
BBF10A4E
                                                                                                                                                 OCTA
                                                                                                                                                 .OCTA
318DAED9
B14AC50E
OF 082899
                                                                                                                                                 .OCTA
                      F7C8ADCD
5B80A780
1FAD866C
5D1512C2
8D5A52DE
4550AA71
4122235B
36F4D031
                                                                                                                                                 .OCTA
                                                                                                                                                 .OCTA
5CB6B1C1
4AA4F5A2
5C864630
47982F45
D7743E13
01A009DF
                                                                                                                                                 .OCTA
                                              FDEEDF5
1B71E0A
                                                                                                                                                 .OCTA
                                                                                                                                                 .OCTA
                                                                                                                                                 .OCTA
                                                                                                                                                 .OCTA
                                                                                                                                                 .OCTA
```

**F

ОТS\$РОШНН 2-006	- REAL*16 ** REAL*16 power routine 16-SEP-1984 02:00:37 VAX/VMS Macro V04-00 Page 6-SEP-1984 11:28:21 [MTHRTL.SRC]OTSPOWHH.MAR;1	(4)
2E21FEC4 397A71D4 62A2AD53 6A6449D8 A83C1DFO D4F8B569 EB345191 9301958C 85427DD4 DA436FD2 OFA04B1F A558EB03 EA951366 B2FBC90B F3BCE667 ACD72EFO 370F3DD2 C5F75E8E DA1F3F6C 51026D7D B018473E 4A01FAB3 F8BA28AC CCE19994 C9BBA192 7C55B5BA AA0D5422 OA23F254 01C34F45 DC5E7B0C 2BE6071F C46B01C7 3F09182A 87BD1CAF 2449C8B4 E2553B23 205A1773 734DD5E8 AD3AF995 3C531AB5 7B086EAA B5E46F2F 1BA62A09 OCB1C222 5529BDD8 9DB71E94 3CBD9150 F9060DCE E0DDEB66 A05A725D BA488DCF 291B39ED 8CACEB96 B9B57337 DB3018F5 F73A9858 490DFA2A 62BB7628 F84B0674 E45465B6 00000000 00000000 00000000 52A07BFC 1D910E8D 74D50A3D 00000000 00000000 000000000 52A07BFC 1D910E8D 74D50A3D 00000000 00000000 000000000000000000	4 \$586.001 0150 153	

```
OTS$POWHH
                                                           - REAL*16 ** REAL*16 power routine
                                                                                                                                                                              VAX/VMS Macro V04-00
[MTHRTL.SRC]OTSPOWHH.MAR;1
 2-006
                                                           DECLARATIONS
 00000000 00000000 00000000 00000000
                                                                                                        .OCTA
                                                                                                                      TWO_M112:
 00000000 00000000 00000000 00003F91
                                                                                                        -OCTA
                                                                                                                      *X00000000000000000000000000003F91
                                                                    D23AFDAO 7D0FE177 B82F7652
00000000 00000000 B82F7652
C142AD1E FA23A474 FB41FA1F
                                                                                        C:
C1:
C2:
                                                                                                       OCTA
                                                                                                                      *XD23AFDA07D0FE177B82F765271544008
*X00000000000000000B82F765271544008
                                                                                                        .OCTA
                                                                                                                      ^XC142AD1EFA23A474FB41FA1FC2EE3FD7
                                                                                        LOGTAB:
E5035347 6399DAAF C2E81C2F
596B3BDD 856DD665 9444039D
4DD0503F 260881C7 BEDEBE21
738FF472 EF9D5774 8AA1F2A6
F2FF977E 57E375D5 D52E256F
AA0C7A7A 32878429 A04E0187
84CFA731 F47D8988 C5E241FA
BA24049D A0F24704 C83B3FFA
00000000 00000000 00000000
                                                 5E423F84
9D1C3F93
F00E3FA2
310C3FB2
84023FC1
03953FD1
7A333FE0
47FD3FF0
00000000
                                                                                                                     .OCTA
                                                                                                                                                                                                  D14
D12
D10
                                                                                                        .OCTA
                                                                                                        .OCTA
                                                                                                        .OCTA
                                                                                                        .OCTA
                                                                                                                                                                                                  08
                                                                                                        .OCTA
                                                                                                                                                                                                  06
                                                                                                                                                                                                  D4
D2
                                                                                                        .OCTA
                                                                                                        .OCTA
                                                                                                                      DO
                                                                                                       .OCTA
                                                                                        LOGLEN = <.-LOGTAB>/16
2EE6E17A 56875E2B C298650F
214A5E47 57994C0F 1BB2C735
769A952E DA1D9F6F B8EC5158
F9095A4F A70E3DA4 5E7C3D39
77C506DA FFE63FD8 5C82223A
8E69C87D 151C686B 8B0CFC58
2342986B B0A876F4 6C7812F8
4BEC9A51 17F61107 A673FE78
EEAC0B53 CBBE729C A4E7B6FB
2BD32BB3 3A76F8B3 4A0B8D70
973606EC F16BEA86 2C58DFF8
07E66730 93C7F357 A39E2FEF
000000000 00000000 00000000
                                                                                        EXPTAB:
                                                                                                                    C3BD3F95
E8CA3F9F
E4CF3FA9
B5253FB3
62CO3FBD
                                                                                                       .OCTA
                                                                                                        .OCTA
                                                                                                        .OCTA
                                                                                                        .OCTA
                                                                                                        .OCTA
                                                 FF CB3F C6
43093FD0
5D873FD9
3B2A3FE2
C6B03FEA
EBFB3FF2
62E43FFA
00000000
                                                                                                        .OCTA
                                                                                                        .OCTA
                                                                                                        .OCTA
                                                                                                        .OCTA
                                                                                                        .OCTA
                                                                                                        .OCTA
                                                                                                        .OCTA
                                                                                                        .OCTA
                                                  0000000D
                                                                                        EXPLEN = <.-EXPTAB>/16
 00000000 00000000 00000000 80004072
                                                                                                                     ^X00000000000000000000000080004072
                                                                                        SHIFT: .OCTA
                                                                                                       .SBTTL OTS$POWHH_R3 - H_floating ** H_floating
                                                                                        FUNCTIONAL DESCRIPTION:
                                                                                                      OTS$POWHH_R3 takes an H_floating (REAL*16) base to an H_floating power and returns an H_floating result in registers RO-R3. This routine is for compiled code support and therefore is not required to follow the
                                                                                                       VAX Procedure Calling Standard.
                                                                                                       The result of the exponentiation is:
                                                                                                       base
                                                                                                                      exponent
                                                                                                                                                   result
                                                                                                       = 0
                                                                                                                      > 0
                                                                                                                                                   0.0
```

0TS

= 0 Undefined Exponentiation Undefined Exponentiation = 0 = Ŏ < 0 Undefined Exponentiation any 2*(exp * log2(base)) > 0 = 0 2*(exp * log2(base))

Floating Overflow can occur.
Floating Underflow can occur.
Undefined Exponentiation can occur if:

1) base is 0 and exponent is 0 or negative base is negative

The basic approach to computing x**y as 2^[y*log2(x)] is the following:

Step 1: Compute log2(x) to sufficient precision to guarantee an accurate final result (see below.)

Step 2: Compute y*log2(x) to at least the accuracy that log2(x)

was computed.

Step 3: Evaluate 2^[y*log2(x)] accurate to the precision of the datatype in question.

To determine the accuracy to which log2(x) must be computed to, write y*log2(x) as I+h, where I is the integer closest to y*log2(x), and h=y*log2(x)-I (Note that !h!=<1/2.) Then

$$2^{(y+\log 2(x))} = 2^{(1+h)} = (2^1)*(2^h).$$

Since the factor 2°I can be incorporated into the final result by an integer addition to the exponent field, we can assume that the multiplication by 2°I incurs no error. Thus the total error in the final result is determined by how accurately 2°h can be computed. If the final result has p fraction bits, we would like h to have at least p good bits. In fact it would be nice if h had a few extra guard bits, say 4. Consequently, we would like h to be accurate to p + 4 bits.

Let e be the number of bits allocated to the exponent field of the data type in question. If I requires more that e bits to represent, then overflow or underflow will occur. Therefore if the product y*log2(x) has e+p+4 good bits, the final result will be accurate. This requires that log2(x) be computed to at least p+e+4 bits.

Since log2(x) must be computed to more bits of precision than is available in the base data type, either the next level of precision or multi-precision arithmetic must be used. We begin by writing

Where c(1) = 1, and $z' = (2/\ln 2)[(z-b)/(z+b)]$. Hence

```
- REAL*16 ** REAL*16 power routine 16-SEP-1984 02:00:37 VAX/VMS Macro V04-00 Page 8 OTS$POWHH_R3 - H_floating ** H_floating 6-SEP-1984 11:28:21 [MTHRTL.SRC]OTSPOWHH.MAR;1 (4)
```

Note that if p(z') is computed to p bits, and log2(b) + z' is computed to p+e+4 bits and overhangs p(z') by e+4 bits, the required accuracy will be achieved. Consequently, the essential tricks, are to pick b such that the overhang can be achieved and to compute log2(b) + z' to p + e + 4 bits.

015

CALLING SEQUENCE:

power.wh.v = OTS\$POWHH_R3 (base.rh.v, exponent.rh.v)

IMPLICIT INPUTS:

OUTPUT PARAMETERS:

IMPLICIT OUTPUTS:

FUNCTIONAL VALUE:

The H floating result is returned in registers RO-R3. This is a violation of the VAX procedure calling standard but is allowed for compiled code support routines.

SIDE EFFECTS:

Modifies registers RO-R3!
MTH\$K_FLOOVEMAT - Floating overflow
MTH\$K_FLOUNDMAT - Floating underflow if FU bit is set
SS\$ ROPRAND - Reserved operand fault
SIGNALS MTH\$ UNDEXP (82 = ' UNDEFINED EXPONENTIATION') if
1) base is 0 and exponent is 0 or negative
2) base is negative

```
0TS
```

```
HHWC9270
600-5
                                                      - REAL*16 ** REAL*16 power routine 16-SEP-1984 02:00:37 0TS$POWHH_R3 - H_floating ** H_floating 6-SEP-1984 11:28:21
                                                                                               .ENTRY OTSSPOWHH_R3, ACMASK
                                                                                     Move x to RO/R3. If x < 0, or x = 0 and y = < 0, return 'UNDEFINED EXPONENTIATION' error condition, otherwise attempt to compute x**y
                                 50 04 AC 70FD
1D 14
07 19
14 AC 73FD
01 15
                                                                                                             #52, SP
base(AP), RO
                                                                                                                                                        Allocate space on the stack R0/R3 < -- x
If x > 0 attempt to compute x**y
                                                                                               SUBL
                                                                                               MOVH
                                                                                               BGTR
                                                                                                             DEFINED
                                                                                               BLSS
                                                                                                                                                         Branch to error code for x < 0
                                                                                                             UNDEFINED
                                                                                                             exp(AP)
                                                                                                                                                         Check sign of y (Note that x = 0)
                                                                                                             UNDEFINED
                                                                                                                                                         Branch to error condition if y =< 0
                                                                                     If processing continues here, this implies that x = 0 and y > 0. Return
                                                                                     x**y = 0
                                                                                               RET
                                                                                                                                                      : Return
                                                                                    If processing continues here, this implies that an undefined exponentiation was attempted. Signal error and return
                                                                                 UNDEFINED:
                                                                                               CLRO
                                                                                                            #AX8000, RO
#MTH$K UNDEXP, -(SP)
#1, G^MTH$$SIGNAL
                                                                                                                                                         RO/R3 <-- Reserved operand
Put error code on stack
                                                                                               MOVW
                                                                                               MOVZBL
                                                                                                                                                        Convert error number to to 32 bit condition code and signal error. NOTE: Second argument is not required since there is no JSB entry.
                       00000000 GF
                                                                                               CALLS
                                                                                               RET
                                                                                    If processing continues here will attempt to compute x**y as 2^{(y*log2(x))}. We begin by determining k and f such that x = 2^k*f, where 1 = 0.
                                                               0699
0699
0699
0699
                                                                                  DEFINED:
                                FFFF8000 8F
00004001 8F
50 54
                                                                                                                                                     : R4 <-- biased exponent of x
: R4 <-- k = exponent of x - 1
: R0 <-- f = 2*(fraction field of x)
                                                                                               BICL3
                                                                                                            #^XFFFF8000, RO, R4
                                                                                                            #^X4001, R4
R4, R0
                                                                                               SUBL
                                                                                               SUBL
                                                               We are now ready to compute log2(x). This computation is based on the
                                                                                     following identity:
                                                                                          \log 2(2^k*f) = k + \log 2(a) + \frac{z}{\ln(2)/2} > \frac{1}{2j+1} z^k(2j+1), where z = \frac{1}{\ln(2)/2} = \frac{1}{2j+1}
                                                                                     We begin by determining a as b^*i, where b=2^*(1/64) and i is 0, 2, 4, ... 64 or 1. Specifically i is chosen by table look-up in such a fashion as to minimize the magnitude of z. Since log2(a)=i/64 we may write
```

```
- REAL*16 ** REAL*16 power routine 16-SEP-1984 02:00:37 OTS$POWHH_R3 - H_floating ** H_floating 6-SEP-1984 11:28:21
                                                                                                                         VAX/VMS Macro VO4-00
[MTHRTL.SRC]OTSPOWHH.MAR:1
                                                                                                                                                                    Page
                                 06AB
06AB
06AB
                                                                               log2(x) = k + 1/64 + z*p(z*z).
                                                    NOTE: for i=2,4,\ldots,64, we may write i=2n, and hence i/64=n/32, i.e. a is an integral power or 2^{6}(1/32). These values are stored in A1 TABLE and A2 TABLE. for i=1, the value of 2^{6}(1/64) is stored immediately BEFORE A1 TABLE and A2 TABLE. Consequently, to access the value of 2^{6}(1/64) from
                                 06AB
                                 06AB
                                 06AB
                                 06AB
                                                     the table, a negative index is used.
                                 06AB
                                 U6AB
                                 06AB
06AB
06AF
06B3
                                                 EVAL_LOG2:
                                                              ROTL
                                                                                                                  R5(0:6) <-- high 7 fraction bits of f
                           78
78
78
78
78
78
78
78
78
78
78
78
78
                                                                                                                 R4 <-- 2^6*k
R5 <-- index to INDEX table
                                                              ASHL
                                                              BICL
  FFFFF93F
                                                                            *INDEX[R5], R5
                                                                                                                  R5 <-- 1 or -1
                                                              CVTBL
                                                              BGEQ
                                                                                                                  Branch if
                                                              INCL
                                                                                                                  R4 <-- 2*6(k + 1/64)
                                                              DECL
                                                                                                                  R5 (-- -2
                                                              BRB
                                                                                                                 Join common code
R5 <-- 2^6*(k + 1/64)
            54
                                                              ADDL
                                                 15:
                                 06CD
                                 06CD
                                                    Since there is no back up data type to compute the necessary guard bits, we proceed by computing z=(f-a)/(f+a) in two parts: z=z1+z2, where z1 is the high 53 bits of z and z2 is the low 113 bits of z. Further, to obtain
                                 06CD
                                 06CD
                                 06CD
                                 06CD
                                                     the desired accuracy it is necessary to work with a = a1 + a2, where a1 and
                                                     a2 are the high and low 113 bits respectively of a. We begin computing (in G-format)
                                 06CD
                                 06CD
                                 06CD
                                 06CD
                                                                                     z1 = (f - a1)/(f + a1)
                                 06CD
                                 06CD
                                                     Note that f-al can be computed exactly in 113 bits, but f+al may require 114
                                                    bits. The 114th bit can be determined by the exclusive or of the low bits of f and al.
                                 06CD
                                 06CD
                                            461
                                 06CD
                                 06CD
  55 FF 8F 78
FFFFF9B6 EF45 7DFD
                                 06CD
                                                                          #-1, R5, R5
L^A1_TABLE[R5], R6
                                                                                                                  R5 <--- octaword offset into A1, A2_TABLE
                                                              ASHL
                                            R6/R9 <-- a1
                                 06D2
                                                              OVOM
                                                                          R9, R3, (SP)
                                                                                                                  SP --> XOR of low bits of al and x
                                 06DB
                                                              XORL3
                                                                                                                        (This will be used to determine the 114th bit of f+al.)
                                 06DF
                                 06DF
                       63FD
61FD
76FD
76FD
46FD
56FD
                                                                          R6. R0. t2(SP)
R6. R0. t4(SP)
t4(SP). R6
t2(SP). R8
04 AE
14 AE
                                 06DF
                                                              SUBH3
           50
50
                                                                                                                      <-- f - a1 (exact)
                   56
56
AE
56
8
                                                              ADDH3
                                                                                                                  t4 < -- f + a1  (rounded)
                                                                                                                  R6/R7 <-- f + a1
                                                              CVTHG
               04
                                                                                                                  R8/R9 <-- f - a1
                                                              CVTHG
                                                                                                                 R8/R9 <-- z1 (G)
t6 <-- z1 (H)
                                                                          R6, R8
R8, t6(SP)
                                                              DIVG2
                                                              CVTGH
                                                     To compute 22 we note
                                                            z = z1 + z2 = (f - a1 - a2)/(f + a1 + a2)
                                 06FE
06FE
06FE
                                                                          z2 = (f - a1 - a2)/(f + a1 + a2) - z1
                                                          let v = f + a1 + a2 = v1 + v2, where v1 and v2 are the high 49 and low
```

113 bits of v respectively. Then

6E

14 AE

50

50

```
- REAL *16 ** REAL *16 power routine 16-SEP-1984 02:00:37 0TS$POWHH_R3 - H_floating ** H_floating 6-SEP-1984 11:28:21
                                                                                                                                                             VAX/VMS Macro V04-00 [MTHRTL.SRC]OTSPOWHH.MAR;1
                                                       z^2 = [(f - a^1 - z^1 * v^1) - (a^2 + z^1 * v^2)]/v
                                                                   We begin by computing v1 and f - a1 - z1*v1
                14 AE
00
56
24 AE
56
                                                                               MOVQ
                                                                                                t4(SP), R6
                                                                                                                                                    R6/R7 <-- high quadword of f + a1
           58
AE
                                                                                                                                                   R6/R9 <-- v1
R0/R3 <-- w - v1 (exact)
R6/R9 <-- z1*v1 (exact)
                                                                                MOVO
                                                                                               #0, R8
                                                                                               R6, t4(SP), R0
t6(SP), R6
R6, t2(SP)
                                                                                SUBH3
                                                                               MULH2
SUBH2
                                                                                                                                                : t2 <-- f - a1 - z1*v1 (exact)
                                                                   Compute v2 and a2 + a1*v2
                                                                                                                                                    Check if w was rounded Branch if not rounded
                                                                               BEGL
                                                                                                #^XFFFEFFF, (SP)
06 13
50 FD9D CF 62FD
FFFFFB84 EF45 60FD
50 24 AE 64FD
FFFFFB76 EF45 60FD
                                                                               SUBHZ
ADDH2
MULH2
ADDH2
                                                                                               TWO M112, RO
L^AZ TABLE[R5], RO
t6(SP), RO
L^AZ_TÁBLE[R5], RO
                                                                                                                                                   Correct for rounding error (exact)
R0/R3 <-- v2
R0/R3 <-- z1*v2
R0/R3 <-- a2 + z1*v2
                                                                   Compute 22
              14 AE 67FD
                                                                               SUBH2
DIVH3
                                                                                               RO, t2(SP)
t4(SP), t2(SP), R6
                                                                                                                                                : t2 <-- (f-a1-z1*v1)-(a2-z1*v2)
: R6/R9 <-- z2
                                                                   The next step is to compute log2(x) accurate to at least 128 bits. This is accomplished as follows, let
                                                                                             w = 2^6*log2(x)

= (2^6)[k + i/64 + z*p(z*z)]

= 2^6(k + i/64) + (2^6)*z*(c0 + c2*z^3 + ... + c10*z^11)

= [2^6*(k + i/64) + z'] + z'(d2*z'^2 + ... + d10*z'^10)

= [2^6*(k + i/64) + z'] + z'*q(z'*z')

= w1 + w2
                                                                  where z'=(2^6*c0)*z and w1 and w2 are the high 49 and low 113 bits of w respectively. Note that the choice of 'a' used in computing z, guarantees that z' overhangs z'*q(z'*z') by at least 15 bits. Hence, if w is computed as w1 + w2, 128 bits of accuracy can be obtained.
                                                                   We begin by defining
                                                                                              c = high 113 bits of (2^6*c0)
c1 = high 49 bits of (2^6*c0)
c2 = low 113 bits of (2^6*c0)
                                                                   then
                                                                                               z' = (z1 + z2)*(c1 + c2)
= z1*c1 + z1*c2 + z2*c.
            FDA4 CF
FD8C CF
FD75 CF
56 50
24 AE
                            65FD
64FD
64FD
61FD
                                                                                               C2, t6(SP), R0
C1, t6(SP)
C, R6
                                                                                                                                                    RO/R3 <-- c2*z1
                                                                               MULH3
                                                                               MULH2
MULH2
ADDH3
                                                                                                                                                    t6 <-- c1*21
                                                                                                                                                   R6/R7 <-- c*z2
t4 <-- c*z2 + c2*z1
R6/R9 <-- z*
```

RO, R6, t4(SP) t6(SP), t4(SP), R6

ADDH3

```
- REAL+16 ** REAL+16 power routine
OTS$POWHH_R3 - H_floating ** H_floating
                                                                                                  16-SEP-1984 02:00:37
6-SEP-1984 11:28:21
                                                                                                                                      VAX/VMS Macro V04-00
[MTHRTL.SRC]OTSPOWHH.MAR;1
                                        543456789
                                                             We proceed by letting
                                                                                    w1 = high 49 bits of <math>2^6*(k + i/32) + z1*c1
                                                             and
                                                                                   w2' = ([2^6*(k + 1/32) + z1*c1 - w1] + z1*c2) + z2*c.
                                                                         ==> 2^6*(k + 1/64) + z' = w1 + w2'
                                                                                      = [2^6*(k + i/64) + z'] + z'*q(z'*z')
= w1 + w2' + z'*q(z'*z')
= w1 + w2,
                                                             where w2 = w2' + z'*Q(z'*z')
                                                                                                                             R4/R5 <-- 2^6(k + i/64)

R2/R3 <-- z1*c1

R0/R1 <-- 2^6(k+i/32) + z1*c1

R2/R3 <-- bits of z1*c1 included in w1

R2/R5 <-- bits of z1*c1 included in w1

L2^6(k+i/32)-w1+z1*c2]
                                                                       CVTLG
                                                                                   t6(SP), R2
R4, R2, R0
R4, R0, R2
R2, R2
R2, t6(SP)
                        AE 54 52 52 AE 50
                             76FD
                                                                       CVTHG
                                                   558
559
560
561
562
563
564
565
                                                                       ADDG3
                              43FD
                                                                       SUBG3
                             56FD
62FD
60FD
56FD
                                                                       CVTGH
      14 AE
04
                AE
                                                                       SUBH2
                                                                                   t6(SP), t4(SP)
RO, t2(SP)
                                                                                                                                  <-- w2'
                                                                       ADDH2
                                                                       CVTGH
                                                   566
567
568
                                                             Compute w2
                        56 65FD
50 75FD
56 64FD
50 60FD
FD64 CF
                                                                                                                             RO/R3 <-- z'*z'
RO/R3 <-- q(z'*z')
RO/R3 <-- z'*Q(z'*z')
                56
08
50
                                                                       MULH3
                                                                                   R6, R6, R0
                                                                      POLYH
                                                                                    RO, #LOGLEN-1, LOGTAB
                                                                                   R6. R0
                                                                      MULH2
ADDH2
            14
                                                                                    RO. t4(SP)
                                                                                                                           : t4 <-- w2
                                                            We now calculate y*log2(x) = (y1+y2)*(w1+w2) = y1*w1 + y2*w1 + y*w2, where y1 and y2 are the high 56 and low 57 bits of y respectively.
                                                   580
581
                   14 AC 70FD
            50
                                                                      MOVH
                                                                                    exp(AP), RO
                                                                                                                           : RO/R3 <-- y
                                       07AA
07AA
07AA
                                                             Test for the possibility of overflow in the computation of yewl.
                                                            This will occur if the exponent of y plus the exponent of w1 is greater than 16383.
        FFFF8000
54 4000
FFFF8000
55 4000
55
55
55
  50
                                       07AA
07B2
07B7
07C0
07C5
07CB
07CD
07CF
07D2
07D2
                                 CB
A2
CB
A2
A0
B1
B1
31
                                                                      BICL3
                                                                                   #^XFFFF8000, RO, R4
                                                                                                                             biased exp of y
                        8F
8F
8F
8F
8F
8F
8F
                                                                      SUBWZ
                                                   588
589
591
593
593
594
597
                                                                                   #^X4000, R4
                                                                                                                              unbiased exp of y
                                                                                                                             biased exp of y
                                                                      BICL3
                                                                                    #^XFFFF8000, t2(SP), R5
                                                                                                                              unbiased exp of y
                                                                       SUBW2
                                                                                    #*X4000, R5
                                                                                   R4, R5
#^x3fff, R5
NO_SYS_OVER_FLOW
Y_TIMES_W1_OVER
                                                                       ADDWZ
                                                                                                                             unbiased exp of w1*y
                                                                       CMPW
                                                                                                                           ; largest unbiased exp possible is 16383
                                                                       BGEQ
                                                                      BRW
                                                         NO_SYS_OVER_FLOW:
                                                                      MUL AZ
            14 AE 54
                             64FD
                                                                                                                          : t4 <-- y*w2
: R4/R5 <-- high 49 bits of y
                                                                                   RO. t4(SP)
                                                                                   RO. R4
                                                                       PVOM
```

```
- REAL+16 ** REAL+16 power routine 16-SEP-1984 02:00:37 VAX/VMS Macro V04-00 0TS$POWHH_R3 - H_floating ** H_floating 6-SEP-1984 11:28:21 [MTHRTL.SRCJOTSPOWHH.MAR;1
015$POWHH
2-006
                                                                                                                                              R4/R6 <-- high 56 bits of y
R4/R7 <-- y1
R0/R3 <-- y2
R0/R3 <-- y2*w1
t2 <-- y1*w1
                                            8F CB
00 D0
54 62FD
AE 64FD
54 64FD
              56
                      52
                                                                                         BICL3
                                                                                                     #^XFFFF01FF, R2, R6
                                                                                                     #0, R7
R4, R0
t2(SP), R0
R4, t2(SP)
                                                                                         MOVL
SUBH2
MULH2
MULH2
                                    50
                                       04
                                                                      601
                                                                               The next step in computing 2^*[y*log2(x)] is to write y*log2(x) as
                                                                                                      y*log2(x) = 1 + j/32 + g/32.
                                                                               where I is an integer, j is an integer between 0 and 31 inclusive, and g is a fraction in the interval L=1/2, 1/2)
                               AE 04 AE 61FD

4 FE61 CF 60FD

4 FE5B CF 62FD

04 AE 54 62FD
                                                                                                     t2(SP), t4(SP), R4
SHIFT, R4
SHIFT, R4
R4, t2(SP)
                   54
                           14 AE
                                                                                         ADDH3
                                                                                                                                             : R4/R7 <-- y1*w1 + y*w2
                                                                                         ADDH2
SUBH2
SUBH2
                                                                                                                                               R4/R7 < -- 2^{6}(1 + 1/32)
                                                                                                                                               t2 <-- those bits of z1*y1 not included in 2*6(1 + j/32)
                                            AE
AE
8F
                                                                                         ADDH2
BICW3
                                                                                                      t2(SP), R0
t4(SP), R0
#^x8000, R4, R8
                                       04
                                50
50
                                                 60FD
                                                                                                                                               RO/R3 <-- 2^7(g/32)
R8 <-- exponent field of 2^6(I+j/32)
R4 <-- 2^5*y*log2(x)
                                                 60FD
                                                    AB
D7
B1
15
                                    8000
                                                                                         DECL
                                                                                                      # X4013, R8
EXCEPTION 1
                                             8F
                                    4013
                                                                                         CMPW
                                                                                         BLEQ
                                                 GAFD
                                                                                                      R4, R8
                                                                                                                                             R8 \leftarrow 2^{5}*(1 + j/32) ir integer
                                                                                         CVTHL
                                                                               We can now compute
                                                                                       x**y = 2^{y*log2(x)} = 2^{1} + \frac{1}{32} + \frac{q}{32}
                                                                                                 = (2^{1})*[A*(B+1)] = 2^{1}*[A + A*B], where
                                                                                A = 2^{\circ}(j/32) is obtained by table look-up and B = 2^{\circ}(g/32) - 1 is obtained
                                                                               by a Min/Max approximation.
                                                                                                     RO, #EXPLEN-1, EXPTAB

#AXFFFFFFFF, R8, R9

A1 TABLELR9], R9

(R9), R0

TABLEN(R9), R0
                             FFFFFFEO 8F CB
                                                                                                                                               R0/R3 < -- B = 2^{(g/32)} - 1

R9 < -- index into A1_TABLE
                                                                                         POLYH
                    FD61 CF
                                                                                         BICL3
                               F853 CF49 7EFD
50 69 64FD
0220 C9 60FD
                                                                                                                                                R9 <-- address of A
                                                                                         DAVOM
                                                                                                                                               RO/R3 <-- A*B

RO/R3 <-- A*B + A2

RO/R3 <-- 2*[(j+g)/32] = (A*B+A2)+A1

R7 = 2*5*1
                                            69
69
1F
                                    50
0220
50
                                                                                         ADDH2
                             50
                                                                                                      (R9), R0
#^X1f, R8
#-5, R8, R8
R8, R0
                                                 60FD
                                                                                         ADDH2
                                                    9C
40
15
                                                                                         BICW
                                             8F
58
08
                                       FB
                               58
                                                                                         ROTL
                                                                                                                                               R0/R3 < -- 2^1*2^[(j+g)/32]
see what exception is if neg or = 0
                                     50
                                                                                                      R8, RO EXCEPTION_2
                                                                                         ADDW
                                                                                         BLEQ
                                                                                         RET
                                                                                                                                               otherwise return result in RO
                                                                                Handlers for software detected over/underflow conditions follow
                                                                            EXCEPTION 1:
                                             50 73FD
                                                                                                      RO
                                                                                                                                           : if big ARG > 0 goto overflow
```

BASI CHF! CHF!

OTS!

EXP EXP MTH MTH MTH MTH OTS

PAR POWI PSL! SETI SF \$I SF \$I

\$QU/ \$\$\$ \$\$\$ \$\$\$ \$\$\$ \$\$\$

UND

PSE

SAB

Phailini Com Pas

Sym Pas Sym Pse Cro

.END

OTS VAX Ass

The 920 The 382

Mac _\$2 148

The

```
E 3
 OTS$POWHH
                                                                                                                       16-SEP-1984 02:00:37 VAX/VMS Macro V04-00 6-SEP-1984 11:28:21 [MTHRTL.SRC]OTSPOWHH.MAR;1
                                                    - REAL*16 ** REAL*16 power routine
                                                                                                                                                                                                                 15 (5)
                                                                                                                                                                                                        Page
 Symbol table
                                                     00000090 R
000003F0
000003F0
00000004 R
000004E0 R
000004F0 R
00000699 R
00000685A R
0000085A R
0000085A R
00000000 R
00000000 R
A1_TABLE
A2_TABLE
ACMASK
                                                                              02
 BASE
                                                   =
                                                                              DEFINED
EVAL LOG2
EXCEPTION 1
EXCEPTION 2
EXP
EXPLEN
 EXPTAB
                                                                              02
 INDEX
LOGLEN
LOGTAB
                                                                              MTH$$SIGNAL
MTHSK_FLOOVEMAT
MTHSK_FLOUNDMAT
MTHSK_UNDEXP
NO_SYS_OVER_FLOW
OTSSPORHH_R3
                                                      ******
                                                      ******
                                                     000007D2 R
00000670 RG
0000087E R
00000004
OVER
SFSW_SAVE_PSW
SHIFT
                                                      00000660
T2
T4
                                                      00000004
                                                      00000014
                                                      00000024
16
                                                     00000220
000004C0 R
00000685 R
TABLEN
TWO_M112
                                                                              20
20
20
20
20
20
20
UNDEF INED
                                                      0000086A R
00000865 R
UNDER
Y_TIMES_W1_OVER
                                                                                 Psect synopsis
PSECT name
                                                                                     PSECT No.
                                                                                                      Attributes
                                                    Allocation
-------
                                                    -------
                                                                                                      NOPIC
NOPIC
PIC
                                                                                                                                                                                    NOWRT NOVEC BYTE NOWRT NOVEC QUAD
     ABS
                                                    00000000
                                                                                                                                      ABS
ABS
REL
                                                                                                                   USR
                                                                                                                             CON
                                                                                                                                                LCL NOSHR NOEXE NORD
$ABS$
                                                                            0.)
                                                    00000000
                                                                                                                   USR
                                                                                                                             CON
                                                                                                                                                LCL NOSHR
LCL SHR
                                                                                                                                                                    EXE
                                                                                                                                                                             RD
 OTS$CODE
                                                    00000892
                                                                                                                   USR
                                                                                                                                                                              RD
                                                                            Performance indicators
                                                                          *-----
Phase
                                                                                         Elapsed Time
                                         Page faults
                                                                 CPU Time
                                                                                         -------
                                                                 00:00:00.10
00:00:00.58
00:00:02.71
00:00:00.05
00:00:01.65
00:00:00.03
                                                                                         00:00:00.84
00:00:02.96
00:00:07.82
00:00:00.07
00:00:05.64
00:00:00.22
Initialization
Command processing
Pass 1
                                                    136
Symbol table sort
                                                    136
Pass 2
Symbol table output
```

**

OTS:

The working set limit was 1200 pages.
13981 bytes (28 pages) of virtual memory were used to buffer the intermediate code.
There were 10 pages of symbol table space allocated to hold 62 non-local and 4 local symbols.
758 source lines were read in Pass 1, producing 17 object records in Pass 2.
9 pages of virtual memory were used to define 8 macros.

! Macro library statistics !

Macro Library name

Macros defined

\$255\$DUA28:[SYSLIB]STARLET.MLB:2

4

88 GETS were required to define 4 macros.

There were no errors, warnings or information messages.

MACRO/ENABLE=SUPPRESSION/DISABLE=(GLOBAL, TRACEBACK)/LIS=LIS\$:OTSPOWHH/OBJ=OBJ\$:OTSPOWHH MSRC\$:MTHJACKET/UPDATE=(ENH\$:MTHJACKET)+MSRC

0265 AH-BT13A-SE

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